Note: I revised the problem sets after I had made the videos. For Problem Sets 1-40, there are gaps in the numbering of the problems (see below) but the problems I solve during the review will have the same number as they do in the problem sets.

Read Ray's New Elementary Algebra, pages 17-22 on Addition.

What is the sum of:

1. 18 and 14 \[32\]

2. \(x\) and 18 \(x + 18\)

6. \(x, z\) and 5 \(x + z + 5\)

What is \(x + 11 + z\) if:

7. \(z = 4\) \(x + 15\)

8. \(x = 22\) and \(z = 5\) \(38\)

10. \(x = y + 1\) \(y + z + 12\)

11. Lisa is 12 years old. How old will she be in:

(a) 5 years \(17\)

(b) \(x\) years \(12 + x\)

(c) \(x\) years, then 4 years, then \(y\) years then 2 years \(18 + x + y\)

12. Thomas has $82 in his savings. How much will he have if he saves an additional:

(a) $12 \($94\)

(b) \(x\) dollars then $5 \($87 + x\)

(c) \(x\) dollars then \(z\) dollars \($82 + x + z\)

What is the difference between:

1. 18 and 14 \( \quad 4 \)
2. \( x \) and 18 \( x - 18 \)
3. \( y \) and 10 \( y - 10 \)

What is \( x - 11 - z \) if:

4. \( z = 4 \) \( x - 15 \)
5. \( x = 22 \) and \( z = 5 \) \( 6 \)
6. \( x = y + 1 \) \( y - 10 - z \)

8. Owen has $47 in his pocket. How much will he have left if he spends:

   (a) $15 and then $12 \( \quad \$20 \)
   (b) \( x \) dollars and then $5 \( \quad \$42 - x \)
   (c) \( z \) dollars then \( x \) dollars then $20 \( \quad \$27 - x - z \)

9. Mr. Flynn is driving toward San Francisco, which is 82 miles away. How far will he be from San Francisco if he drives:

   (a) 16 miles \( \quad 66 \text{ miles} \)
   (b) \( x \) miles \( \quad 82 - x \)
   (c) \( x \) miles, then \( y \) miles \( \quad 82 - x - y \)
   (d) 5 miles, then \( x \) miles, then \( z \) miles, then 2 miles \( \quad 75 - x - z \)
For each of the following subtraction problems, write the correct addition problem:

1.  $5 - 3 = 2$  \quad $2 + 3 = 5$

2.  $17 - x = 8$  \quad $x + 8 = 17$

3.  $x - y = z$  \quad $x = y + z$

Read through section 68 in the section on "Multiplication" in *Ray's Elementary Algebra*, pp. 30 through top of p. 32.

For each of the following addition problems, write the correct multiplication problem:

4.  $6 + 6 + 6 + 6 + 6 + 6$  \quad $6 \times 6$

5.  $x + x + x + x + x + x$  \quad $6x$

6.  $4 + 4 + 4 + x + x + x$  \quad $3 \times 4 + 3x$

13.  $5 + x + y + 5 + y + x + z + z + 5 + x + z$  \quad $5 \times 3 + 3x + 2y + 3z$

Find the products of:

14.  12 and 4  \quad 48

15.  $x$ and 4  \quad $4x$

18.  $x$ and $y$ and $2z$  \quad $2xyz$

Find the product of $6xz$ if:

19.  $x = 2$  \quad $12z$

20.  $x = 2$ and $z = 6$  \quad $72$
What is the sum of:

1. \( y \) and 10 \( y + 10 \)

2. \( x, y, \) and 10 \( x + y + 10 \)

3. \( x, z, \) 3 and 5 \( x + z + 8 \)

4. \( x, b, \) and 11 \( x + b + 11 \)

What is \( x + 4 + y \) if:

5. \( y = 11 \) \( x + 15 \)

6. \( x = 22 \) and \( y = 4 \) \( 30 \)

7. \( x = 1 \) \( y + 5 \)

8. John has driven 27 miles. How far will he have driven if he drives another:
   
   (a) 16 miles \( 43 \)
   
   (b) \( x + 4 \) miles \( x + 31 \)
   
   (c) \( a + b \) miles \( a + b + 27 \)

What is the difference between:

9. 17 and 5 \( 12 \)

10. \( a \) and 24 \( a - 24 \)

11. \( y \) and 105 \( y - 105 \)

What is \( x - y - 4 \) if:

12. \( y = 11 \) \( x - 15 \)

13. \( x = 16 \) and \( y = 4 \) \( 8 \)

14. \( x = 5 \) \( 1 - y \)
15. Sam is 7 years younger than his sister. How old will Sam be when his sister is:

(a) 28  21
(b) $x + y$  $x + y - 7$
(c) 14 + $b$  $b + 7$

For each of the following subtraction problems, write the correct addition problem:

16. $17 - 9 = 8$  $17 = 8 + 9$
17. $x - y = 16$  $x = 16 + y$
18. $x - 4 = 11$  $x = 11 + 4$

For each of the following addition problems, write the correct multiplication problem:

19. $2 + 2 + 2 + x + x + z + z + z$  $2 \times 3 + 2x + 3z$
20. $y + y + y + y + x + x$  $2x + 4y$

Find the products of:

21. $x$ and 4 and 6  $24x$
22. $x$ and 4$y$  $4xy$
23. $4ab$ if $a = 3$ and $b = x$  $12x$
24. $12xy$ if $x = 2$ and $y = 3$  $72$
Solve or write the following division problems:

1. The quotient of 12 and 6 2

4. The quotient of \( x \) and \( y \) \( \frac{x}{y} \)

Rewrite the following division problems as multiplication problems:

5. \( \frac{x}{10} = 8 \)  \( x = 8 \times 10 \)

6. \( \frac{24}{x} = 12 \)  \( 24 = 12x \)

10. \( \frac{432}{4} = 108 \)  \( 432 = 4 \times 108 \)

11-14. Mr. Sandberg built 6 model trains with 20 cars each.

(a) If his cost was $360, what was his cost for each train?  $60

(b) If his cost was $360, what was his cost for each car?  $3

(c) If his cost was \( a \) dollars, what was his cost for each train?  \( \frac{a}{6} \)

(d) If his cost was \( a \) dollars, what was his cost for each car?  \( \frac{a}{120} \)

Find the missing dimension for each rectangle (the numbers inside are the area):

14.  

\[
\begin{array}{c|c|c}
9 & 72 & 8 \\
\end{array}
\]
15. 

A cube has six faces.

18. 

19. If each face of a cube has 16 square inches, what is its total surface area?
   96

20. If each face of a cube has a surface area of $b$ square inches, what is its total surface area?
   $6b$

21. If the total surface area of a cube is $y$ square inches, what is the surface area of one face?
   $y/6$
Find each of the following powers:

1. $6^2$  
   $36$
2. $4^3$  
   $64$
3. $10^7$  
   $10,000,000$
8. $(0.5)^3$  
   $0.125 = 1/8$

Write a multiplication equivalent to the following powers:

9. $8^6$  
   $8 \times 8 \times 8 \times 8 \times 8 \times 8$
10. $x^8$  
   $x \times x \times x \times x \times x \times x \times x \times x$
11. $4^y$  
   $4 \times 4 \times 4 \times \ldots \times (x \text{ times})$
12. $x^y$  
   $x \times x \times \ldots \times (y \text{ times})$

13. Make a table showing the first ten powers of two.

<table>
<thead>
<tr>
<th></th>
<th>2^1</th>
<th>2^2</th>
<th>2^3</th>
<th>2^4</th>
<th>2^5</th>
<th>2^6</th>
<th>2^7</th>
<th>2^8</th>
<th>2^9</th>
<th>2^10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
</tr>
</tbody>
</table>

14. 625 is what power of 5? 4th

15. 243 is what power of 3? 5th

16. 256 is what power of 4? 4th
Evaluate each of the following expressions:

1. \[ 9 - 3(8 - 6) = 3 \]
2. \[ 5 + 6(5 - 3) = 17 \]
3. \[ 25 - 4(7 - 2) = 5 \]
4. \[ 6 \cdot 6 + 9 \cdot 9 = 117 \]
5. \[ 31 + 3[16 \div (2 \cdot 4)] = 37 \]
6. \[ 29 - 2[12 \div (2 \cdot 3)] = 25 \]
7. \[ 84 - 2(3^4 - 51) = 24 \]
8. \[ 51 - 3(2^5 - 22) = 21 \]
9. \[ 33 - 3[4 \cdot (7 - 5)] + 3^2 = 18 \]
10. \[ 4 + 4[3 \cdot (9 - 4)] - 5^2 = 39 \]
Use the distributive rule to write each of the following expressions as a sum or difference:

1. \(3(x - 6) = 3x - 18\)
2. \(4(x + 2) = 4x + 8\)
3. \((x - 4 + 2)10 = 10x - 20\)
4. \(3a(x - y + 1) = 3ax - 3ay + 3a\)
5. \(3ab(x - y - z + a + b + c) = 3abx - 3aby - 3abz + 3a^2b + 3ab^2 + 3abc\)

Write the opposites and the absolute values of the following numbers:

6. \(4\); \(-4\); \(|4| = 4\)
7. \(-24\); \(24\); \(|-24| = 24\)
8. \(-2\); \(2\); \(|-2| = 2\)
17. \(13\); \(-13\); \(|13| = 13\)
18. \(-108\); \(108\); \(|-108| = 108\)
Solve or write the following division problems:

1. Six divided by $x$ \[ \frac{6}{x} \]

2. The quotient of $x$ and 7 \[ \frac{x}{7} \]

Rewrite the following division problems as multiplication problems:

3. \[ \frac{40}{5} = x \]

4. \[ \frac{25}{x} = 5 \]

5. \[ \frac{24}{x} = 12 \]

\[ 40 = 5x \]
\[ 25 = 5x \]
\[ 24 = 12x \]

Find the missing dimension for the rectangle (the number inside is the area):

6. \[ \frac{x}{15} \]

Find the following powers:

7. $2^4$ = 16

8. $12^3$ = 1728

9. $1^8$ = 1

10. $0^{21}$ = 0

11. Write a multiplication equivalent to $y^5$ \[ y \cdot y \cdot y \cdot y \cdot y \]

12. 729 is what power of 9? \[ 3^{rd} \]

13. 729 is what power of 3? \[ 6^{th} \]
Evaluate each of the following expressions using the correct order of operations:

14. \[ 8 + 3 \cdot 4^3 = 200 \]

15. \[ 7 + 2(18 - 5) = 33 \]

16. \[ 2 + 8 \cdot 3^2 = 74 \]

17. \[ 33 - 3 \cdot 2^3 = 9 \]

18. \[ 34 - 4 \cdot 2^3 = 2 \]

19. \[ 14 + 4(7 - 4 + 1) = 30 \]

20. \[ 19 + 3(8 - 5 + 2) = 34 \]

21. \[ 12 + 10 ÷ 2 - 1 = 16 \]

22. \[ 25 + 99 ÷ 3 - 8 = 50 \]

23. \[ 5 \cdot 4 + 7 \cdot 8 = 76 \]

Use the distributive rule to write each of the following expressions as a sum or difference:

24. \[ a(x - 9) = ax - 9a \]

25. \[ y(x + y) = y^2 + xy \]

26. \[ b(8 - 4) = 4b \]

27. \[ 2c(9 - 2) = 14c \]

28. \[ x(x + y + z) = x^2 + xy + xz \]

Write the opposites and the absolute values of the following numbers:

29. \[ -\frac{1}{2}; \quad \frac{1}{2}; \quad |\frac{1}{2}| = \frac{1}{2} \]

30. \[ 116; \quad -116; \quad |116| = 116 \]

31. \[ 67,871; \quad -67,871; \quad |67,871| = 67,871 \]

32. \[ -257; \quad 257; \quad |\text{-257}| = 257 \]

33. \[ 1; \quad -1; \quad |1| = 1 \]
Evaluate each of the following expressions:

6. \(4 + (-9) = -5\)

7. \(-4 + 9 = 5\)

8. \(-9 + 4 = -5\)

9. \(5.6 + (-4.2) = 1.4\)

10. \(|-7| = 7\)

11. \((-(-7)) = 7\)

12. \(-7 + 7 = 0\)

13. \(-7 + (-7) = -14\)

14. \(-(-(-7))) = 7\)

15. \(-6\frac{4}{5} + \frac{2}{5} = \frac{-32}{5}\)

30. \(35 - 21 - 12 + 18 - (-14) - 48 = -14\)
Evaluate each of the following expressions:

8. $-9 \cdot 4 = -36$

9. $5.6 \cdot (-4.2) = -23.52$

10. $5 \cdot |7| = 35$

11. $(-1)(-7) = 7$

12. $-7 \cdot 7 = -49$

13. $-7 \cdot (-7) = 49$

14. $-1(-1(-1(-7))) = 7$

15. $\frac{-4}{5} \cdot \left(\frac{2}{5}\right) = \frac{-8}{25}$

16. $(-2)^4 = 16$

17. $(-2)^5 = -32$

18. $-2^4 = -16$

19. $-2^5 = -32$

30. In your summary notebook, write a neat summary of the operations with 0 and 1 for addition, subtraction, multiplication, and division.
Solve each of the following equations. Be sure to show the transformation:

1. \( x + 6 = 10 \)  \( x = 10 - 6 = 4 \)

2. \( x - 76 = 152 \)  \( x = 152 + 76 = 228 \)

3. \( x - 1979 = 2013 \)  \( x = 2013 + 1979 = 3992 \)

4. \( \frac{1}{4} x = 28 \)  \( x = 4 \times 28 = 112 \)

5. \( 4x = 28 \)  \( x = 28/4 = 7 \)

6. \( \frac{3}{4} x = 6 \)  \( x = 6 \times 4/3 = 8 \)

7. \( \frac{3}{5} x = 27 \)  \( x = 27 \times 5/3 = 45 \)

8. \( \frac{5}{2} x = -25 \)  \( x = -25 \times 2/5 = -10 \)

9. \( x + \frac{3}{4} = \frac{5}{4} \)  \( x = \frac{5}{4} - \frac{3}{4} = \frac{1}{2} \)

10. \( x - \frac{3}{4} = 1\frac{3}{4} \)  \( x = 1\frac{3}{4} + \frac{3}{4} = 2 \frac{1}{2} \)

11. \( \frac{3}{2} x = -18 \)  \( x = -18 \times 2/3 = -12 \)

12. \( \frac{4}{5} x = 24 \)  \( x = 24 \times 5/4 = 30 \)
1. The length of a rectangle is 7 cm more than its width. Let \( x \) be the number of cm in the width.

   a. Draw the rectangle. Then write an expression for its length. \( L = x + 7 \)
   b. Find the length if the width is: i. 12 cm;   ii. 37 cm;   iii. 100 cm.

      \[
      \begin{array}{c}
      \text{19 cm} \\
      \text{44 cm} \\
      \text{107 cm}
      \end{array}
      \]

2. Susan is 5 years younger than her sister Katie. Let \( x \) be Katie's age.

   a. Write an expression for Susan's age. \( \text{Susan's age} = x - 5 \)
   b. How old will Susan be when Katie is: i. 17;   ii. 23;   iii. 100?

      \[
      \begin{array}{c}
      \text{12} \\
      \text{18} \\
      \text{95}
      \end{array}
      \]
   c. Write an equation stating that Susan is 24. Then solve the equation to find Katie's age. \( 24 = x - 5; \quad x = 24 + 5 = 29 = \text{Katie's age} \)
   d. Repeat the procedure in part (c) to find Katie's age when Susan is i. 13   ii. 72

      \[
      \begin{array}{c}
      \text{i. Katie's age} = 13 + 5 = 18 \\
      \text{ii. Katie's age} = 72 + 5 = 77
      \end{array}
      \]

3. When you travel downstream, your actual speed is your speed through the water plus the speed of the current. The San Antonio River flows at about 3 kilometers per hour (km/h). Let \( x \) be the number of kilometers per hour you go through the water.

   a. Write an expression representing your actual speed downstream. \( \text{Actual speed} = 3 + x \)
   b. How fast would you go downstream in:
      i. a pedalboat, which goes 5 km/h;   \( 8 \text{ km/h} \)
      ii. a rowboat, which goes 11 km/h;   \( 14 \text{ km/h} \)
      iii. a speedboat, which goes 42 km/h? \( 45 \text{ km/h} \)
   c. Write an equation stating that your actual speed is 21 km/h. Then solve the equation to find your speed through the water. \( 21 = 3 + x; \quad x = 21 - 3 = 18 \)
   d. Repeat the procedure in part (c) to find your speed through the water if your actual speed is: i. 9 km/h;   ii. 65 km/h

      \[
      \begin{array}{c}
      \text{x = 9 - 3 = 6} \\
      \text{x = 65 - 3 = 62}
      \end{array}
      \]
Evaluate each of the following expressions:

1. \( 9 + (-5) = 4 \)
2. \( 8 + (-13) = -5 \)
3. \( -2 + (-16) = -18 \)
4. \( -4 + 11 = 7 \)
5. \( -21 + 7 = -14 \)
6. \( 24 - 150 = -126 \)
7. \( 12 - (-15) = 27 \)
8. \( -11 - (-2) = -9 \)
9. \( -99 - 59 = -158 \)
10. \( -212 - (-37) = -175 \)
11. \( 9 \cdot (-5) = -45 \)
12. \( 8(-13) = -104 \)
13. \( -2 \cdot (-16) = 32 \)
14. \( -5 \cdot 11 = -55 \)
15. \( -21(7) = -147 \)
16. \( 4 \cdot (-9) = -36 \)
17. \( -4 \cdot 9 = -36 \)
18. \( -212 \cdot (-37) = 7844 \)
19. \( 52 \div (-4) = -13 \)
20. \( 12 \div (-6) = -2 \)
21. \(0 \div (-12) = 0\)

22. \(\frac{-5}{6} + \frac{-1}{6} = 5\)

Solve each of the following equations. Be sure to show the transformation:

23. \(14x = -28\) \(x = -28/14 = -2\)

24. \(\frac{1}{14}x = -28\) \(x = -28 \times 14 = -392\)

25. \(10x = 2\) \(x = 2/10 = 1/5\)

26. \(x + 12 = 19\) \(x = 19 - 12 = 7\)

27. \(x - 74 = -418\) \(x = -418 + 74 = -344\)

28. \(x + 1054 = 1517\) \(x = 1517 - 1054 = 463\)

29. \(\frac{1}{3}x = 57\) \(x = 57 \times 3 = 171\)

30. About one-fifth of all people are left-handed. Let \(x\) be the number of people in a particular group.

   a. Write an expression for the number of left-handed people in the group.
      \[\text{number} = \frac{x}{5}\]
   
   b. Write an equation stating that the number of left-handers in a group is 47. Solve the equation to find the number of people in that group.
      \[47 = \frac{x}{5}\] \(x = 47 \times 5 = 235\)
   
   c. Find the number of people in groups that have:
      i. 100 left-handers; \(x = 100 \times 5 = 500\)
      ii. 4000 left-handers \(x = 4000 \times 5 = 20000\)
   
   d. How many left-handers are there in a group of:
      i. 30 people; \(\text{number} = \frac{x}{5} = 30/5 = 6\)
      ii. 100 people; \(\text{number} = \frac{x}{5} = 100/5 = 20\)
      iii. 5000 people? \(\text{number} = \frac{x}{5} = 5000/5 = 1000\)

31. A construction company’s rules say that a supervisor earns 1.5 times as much as a laborer. Let \(x\) be the number of dollars per hour that a laborer earns.

   a. Write an expression for the number of dollars per hour a supervisor earns.
      \[\text{supervisor wage} = 1.5x\]
b. Doug is a laborer who earns $12.00 per hour. How much does his supervisor earn?  
\[ 1.5 \times 12 = 18 \]

c. If Doug’s pay is increased to $16.00 per hour, how much will the supervisor earn? 
\[ 1.5 \times 16 = 24 \]

d. Suppose that a supervisor earns $19.20 per hour. How much does the laborer earn under these conditions?  
\[ 19.2 / 1.5 = 12.8 \]

32. The number of seconds it takes the thunder sound to reach you is 3 times the number of kilometers between you and the lightning. Let \( x \) be the number of kilometers.

a. Write an expression for the number of seconds the sound takes to reach you.

b. How long does it take the sound to reach you if the lightning is:
   i. 5 kilometers away?  
   \[ \text{seconds} = 5 \times 3 = 15 \]
   ii. 2.8 kilometers away?  
   \[ \text{seconds} = 2.8 \times 3 = 8.4 \]

c. Write an equation stating that the sound takes 12 seconds to reach you. Then solve it to find your distance from the lightning. 
\[ 3x = 12; x = 4 \text{ km} \]

33. When money is left in a savings account, it earns “simple interest”. If the interest rate is 10% per year, you will have 1.1 times as much money at the end of 1 year as you had to start with. Each year the amount is multiplied by 1.1 again. If you start with \( x \) dollars, how much will you earn at the end of 30 years? Write the answer as a power.

\[ x \cdot 1.1^{30} \]
Factor out the common factor(s):

1. \(5x + 5y = 5(x + y)\)

2. \(3a - 3b = 3(a - b)\)

3. \(8c - 16 = 8(c - 2)\)

4. \(14x - 7 = 7(2x - 1)\)

5. \(6x + 8y = 2(3x + 4y)\)

6. \(3ab - 6ac = 3a(b - 2c)\)

7. \(4ax - 12bx = 4x(a - 3b)\)

8. \(x^2 - 9x = x(x - 9)\)

9. \(7y^2 + 14y = 7y(y + 2)\)

10. \(3x + 3y - 6z = 3(x + y - 2z)\)

11. \(\frac{x}{3} + \frac{1}{3} = (x + 1)/3\)

12. \(x^3 + x^2 = x^2(x + 1)\)
Solve the following equations. Then check your answer by substituting the value of the variable back into the original equation.

1. \(-\frac{1}{4}x - 9 = 17\) \(x = -104\)
2. \(13 - 4y = 25\) \(y = -3\)
3. \(8 - \frac{1}{7}v = -19\) \(v = 189\)
4. \(53 = 5x + 11\) \(x = \frac{42}{5} = 8.4\)
5. \(17 - x = 25\) \(x = -8\)
9. \(-21 + 4p = 10\) \(p = \frac{31}{4} = 7.75\)

12. Hildy, a German Shepherd, weighs 97 pounds. She goes on an eating spree, gaining 1/4 pound per day.

a. Write an expression representing her weight after \(x\) days.
   \[\text{weight after } x \text{ days} = 97 + \frac{x}{4}\]

b. How much will she weigh after:
   i. 12 days \(100\) pounds
   ii. 4 weeks? \(104\) pounds

c. How long will it take her to reach:
   i. 150 pounds \(212\) days
   ii. 160 pounds \(252\) days

13. The temperature inside the earth is assumed to increase by about 10 degrees Celsius for every kilometer beneath the surface. Suppose that the temperature at the surface is 24ºC. Let \(x\) be the number of kilometers beneath the surface.

a. Write an expression for the number of degrees at a depth of \(x\) kilometers.
   \[\text{Degrees } C \text{ at depth of } x \text{ km} = 24 + 10x\]

b. Find the temperature inside a coal mine 1.3 kilometers deep.
   \[37\] degrees C

c. Find the temperature at the bottom of an oil well 5 kilometers deep.
   \[74\] degrees C

d. Write an equation stating that the temperature at the bottom of a diamond mine is 61ºC. Then solve the equation to find the depth of the mine.
   \[61 = 24 + 10x; \quad x = 3.7 \text{ km}\]
e. At what depth would water boil (100ºC)?
   \[100 = 24 + 10x; \quad x = 7.6 \text{ km}\]

1-5. Solve exercises 6-10 on p. 95.

6) \[ \frac{x}{a} + \frac{x-5}{2} = b \quad \Rightarrow \quad 2a \left( \frac{x}{a} + \frac{x-5}{2} \right) = 2a \cdot b \quad \Rightarrow \quad 2x + ax - 5a = 2ab \]

7) \[ \left[ 12(x-3) \right] \left( \frac{\frac{1}{2} + \frac{2a}{3}}{x-3} \right) = \left( \frac{3}{4} \right) \left[ 12(x-3) \right] \quad \Rightarrow \quad 48 + 8a(x-3) = 9(x-3) \]

8) \[ (x-3)(a-b) \left( \frac{x+1}{x-3} + \frac{3-c}{a-b} \right) = (a)(x-3)(a-b) \]
\[ \Rightarrow (a-b)(x+1) + (x-3)(3-c) = a(x-3)(a-b) \]

9) **Note:** \( (a+b)(a-b) = a^2 - b^2 \)
\[ (a-b) \left( \frac{x}{a+b} + \frac{b}{a-b} \right) = \left( \frac{c}{a^2-b^2} \right) \left( a^2-b^2 \right) \quad \Rightarrow \quad x(a-b) + x(a+b) = c \]

10) \[ b d f x \cdot \left( \frac{a}{b x} + \frac{c}{d x} + \frac{e}{f x} \right) = \left( h \right) \cdot b d f x \]
\[ \Rightarrow a d f + c b f + e b d = h b d f x \]
Pages 100-106 show many examples worked in detail. If you have any trouble with the following problems, read through pages 100-106.


18) \[a + b + c = 60\]
   \[b = 3a\]
   \[c = 2b = 6a\]
   \[\Rightarrow a + 3a + 6a = 10a = 60 \Rightarrow \{a = 6\}\]
   \[b = 3a = 3 \cdot 6 = 18 \Rightarrow \{b = 18\}\]
   \[c = 2b = 6a = 6 \cdot 6 = 36 \Rightarrow \{c = 36\}\]
   \[\Rightarrow c + 18 + 3c = 60\]

19) apples \times 1 + lemons \times 2 + oranges \times 5 = 56

   apples = lemons = oranges = \(x\)
   \[\Rightarrow x + 2x + 5x = 56 = 8x \Rightarrow \{x = 7\}\]
   \[\Rightarrow \text{bought 7 each, apples, lemons, oranges}\]

20) See review video

21) \[A = 2B\]
   \[B = 2C \Rightarrow A = 4C\]
   \[A + B + C = 98 \Rightarrow 4C + 2C + C = 7C = 98 \Rightarrow C = \frac{98}{7}\]
   \[\Rightarrow \{C = 14, A = 56, B = 28\}\]
   \[14 + 56 + 28 = 98\]

22) \[A + B + C + D = 44\]
   \[B = 3A, \ C = A, \ D = B + C = 3A + A = 4A\]
   \[\Rightarrow A + 3A + A + 4A = 9A = 44 \Rightarrow \{A = \frac{44}{9}\}\]
   \[D = \frac{132}{9}\]
   \[B = \frac{44}{3}\]
   \[C = \frac{39}{9}\]
   \[D = \frac{176}{9}\]
For each of the following equations, name the axiom, property or definition that is being used:

1. $3(50 + 90) = (50 + 90)3$  \hspace{1cm} \textit{commutative axiom of multiplication}
2. $3(50 + 90) = 3(90 + 50)$  \hspace{1cm} \textit{commutative axiom of addition}
3. $3(50 + 90) = 3 \cdot 90 + 3 \cdot 50$  \hspace{1cm} \textit{distributive axiom}
4. $3(50 \cdot 90) = (3 \cdot 50) \cdot 90$  \hspace{1cm} \textit{associative axiom of multiplication}
5. $3 + (50 + 90) = (3 + 50) + 90$  \hspace{1cm} \textit{associative axiom of addition}
6. $3 - 50 = -47$, and $-47 = -40 - 7$, so $3 - 50 = -40 - 7$  \hspace{1cm} \textit{transitive axiom of equality}
7. $-1 \cdot 3 = -1 \cdot 3$  \hspace{1cm} \textit{reflexive axiom of equality}
8. $0 \cdot x = x \cdot 0$  \hspace{1cm} \textit{commutative axiom of multiplication}
9. Since $3x + 5x = 8x$, $8x = 3x + 5x$  \hspace{1cm} \textit{symmetric axiom of equality}
10. $3x + 5x = (3 + 5)x$  \hspace{1cm} \textit{distributive axiom}
11. If $3x = 5x + 7$, then $3x + (-5x) = 5x + 7 + (-5x)$  \hspace{1cm} \textit{addition property of equality}
12. If $\left(\frac{2}{3}\right)x = 12$ then $\left(\frac{2}{3}\right)x \cdot \left(\frac{3}{2}\right) = 12 \cdot \frac{3}{2}$  \hspace{1cm} \textit{multiplication property of equality}
13. If $x = 7$ and $7 = z$, then $x = z$  \hspace{1cm} \textit{transitive axiom of equality}
14. $-y + y = 0$  \hspace{1cm} \textit{additive inverses axiom}
15. $\left(\frac{1}{z}\right) \cdot z = 1$  \hspace{1cm} \textit{multiplicative inverses axiom}
Factor out the common factors:

1. \(12a + 4 = 4(3a + 1)\)
2. \(5x^2 + 2x = x(5x + 2)\)
3. \(4x + 8y + 12z = 4(x + 2y + 3z)\)
4. \(3ax + 18ay = 3a(x + 6y)\)
5. \(x^2y + xy^2 = xy(x + y)\)

Solve the following equations. Then check your answer by substituting the value of the variable back into the original equation.

6. \(71 = 4 - x\) \(x = -67\)
7. \(\frac{2}{3}x + 6 = 18\) \(x = 18\)
8. \(0.9 + 3x = 6\) \(x = 1.7\)
9. \(17 - x = 25\) \(x = -8\)
10. \(-\frac{4}{3} - x = -\frac{1}{3}\) \(x = -1\)

11. Donald’s Donuts sells donuts for 20 cents each, plus 15 cents for the box in which they come. So the total number of cents you pay is 20 times the number of donuts, plus 15. Let \(x\) be the number of donuts.

   a. Write the definition of \(x\) on your paper. Then write an expression for the number of cents you pay for \(x\) donuts. \(x: number\ of\ donuts\ cost = 20x + 15\)

   b. How much will you pay for:  
      i. 12 donuts \(\$2.55\)  
      ii. 100 donuts? \(\$20.15\)

   What assumption must you make about the box in order for (ii) to be reasonable?  
   \(100\ donuts\ must\ fit\ in\ a\ single\ box\)
c. Write an equation stating that the number of cents you pay is 355. Then solve the equation to find out how many donuts you get for $3.55.

\[
\text{cost} = 20x + 15 = 355 \\
x = 17
\]

12. Doug must shovel a pile containing 50 cubic feet ($\text{ft}^3$) of sand into a dump truck. With each scoop, he decreases the size of the pile by $\frac{1}{6}$ $\text{ft}^3$. Let $x$ be the number of scoops he has shoveled.

a. Write the definition of $x$. Then write an expression for the number of cubic feet of sand left in the pile after $x$ scoops.  

\[
x: \text{number of scoops shoveled} \\
\text{sand left} = 50 - x/6
\]

b. How much sand is left after:  
   i. 12 scoops;  
   ii. 100 scoops?

\[
50 - 12/6 = 48 \\
50 - 100/6 = 33 \frac{1}{3}
\]

c. Doug takes a rest when 20 $\text{ft}^3$ of sand remain. Write an equation stating that 20 $\text{ft}^3$ remain. Then solve the equation to find out how many scoops Doug has shoveled before he rests.

\[
20 = 50 - x/6 \\
x = 6(50 - 20) = 180
\]

13-17. In Ray’s New Elementary Algebra, solve problems 1-5 on p. 95.

1) \[ \frac{x}{2} + \frac{x}{3} = 5 \Rightarrow 6\left(\frac{x}{2} + \frac{x}{3}\right) = 6 \cdot 5 \Rightarrow 3x + 2x = 30 \]

2) \[ \frac{x}{3} - \frac{x}{4} = 2 \Rightarrow 12\left(\frac{x}{3} - \frac{x}{4}\right) = 2 \cdot 12 \Rightarrow 4x - 3x = 24 \]

3) \[ \frac{x}{4} + \frac{x}{8} - \frac{x}{6} = \frac{5}{12} \Rightarrow 24\left(\frac{x}{4} + \frac{x}{8} - \frac{x}{6}\right) = \frac{5 \cdot 24}{12} \Rightarrow 6x + 3x - 4x = 10 \]

4) \[ 2\left(\frac{2x - 3}{4} + \frac{x}{7}\right) = \left(\frac{x - 3}{2} + \frac{5}{14}\right) \Rightarrow 7(2x-3) - 4x = 14(x-3) + 10 \Rightarrow 18x - 21 = 14x - 32 \]

5) \[ x - \frac{x - 3}{2} = 5 - \frac{x + 4}{3} \Rightarrow 6(x - \frac{x - 3}{2}) = 6(5 - \frac{x + 4}{3}) \Rightarrow 6x - 3x - 9 = 30 - 2x + 8 \]
For the given function, fill out the values of \( y \) for the listed values of \( x \).

1. \( y = 3x \)
   \[
   x: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
   y: \quad 0 \quad 3 \quad 6 \quad 9 \quad 12 \quad 15
   \]

2. \( y = 4x - 3 \)
   \[
   x: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
   y: \quad -3 \quad 1 \quad 5 \quad 9 \quad 13 \quad 17
   \]

3. \( f(x) = x^2 \)
   \[
   x: \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \\
   f(x): \quad 9 \quad 4 \quad 1 \quad 0 \quad 1 \quad 4 \quad 9
   \]

4. \( f(x) = \frac{12}{x} \)
   \[
   x: \quad 1 \quad 2 \quad 3 \quad 4 \quad 6 \quad 8 \\
   f(x): \quad 12 \quad 6 \quad 4 \quad 3 \quad 2 \quad 3/2
   \]

7. \( f(x) = 2x^2 - x + 3 \)
   \[
   x: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
   f(x): \quad 3 \quad 4 \quad 9 \quad 18 \quad 31 \quad 48
   \]

8. \( y = -x^3 \)
   \[
   x: \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \\
   y: \quad 27 \quad 8 \quad 1 \quad 0 \quad -1 \quad -8 \quad -27
   \]

For the following pairs of numbers in the tables, write down the formula for a function that gives the same results. You may write the function using \( y = \) or \( f(x) = \).

9. \[
   \begin{array}{cccccc}
   x: & 0 & 1 & 2 & 3 & 4 & 5 \\
   y: & 0 & 4 & 8 & 12 & 16 & 20
   \end{array}
   \quad f(x) = 4x
   \]

10. \[
    \begin{array}{cccccc}
    x: & 0 & 1 & 2 & 3 & 4 & 5 \\
    y: & -5 & -4 & -3 & -2 & -1 & 0
    \end{array}
    \quad f(x) = x - 5
    \]

11. \[
    \begin{array}{cccccc}
    x: & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    y: & -27 & -8 & -1 & 0 & 1 & 8 & 27
    \end{array}
    \quad f(x) = x^3
    \]
12. Evan earns $80 per week in the summer mowing lawns. Write a formula that represents how much Evan has earned in the summer after $t$ weeks. Use the formula $y = \ldots$. Identify the dependent and the independent variables in this function.

\[ y = 80t \quad ; \quad y \text{ is dependent variable; } t \text{ is independent variable} \]

15. Katie saved $600 during the summer. If she spends $20 per week and does not earn any more money, how much money will she have after $t$ weeks? Write a formula that represents this function. Use the formula $y = \ldots$. Identify the dependent and the independent variables in this function.

\[ y = 600 - 20t \quad y \text{ is dependent variable; } t \text{ is independent variable} \]
For the given function, fill out the values of $y$ for the listed values of $x$ and then graph the function on graph paper. Be sure to draw the coordinate axes and label them correctly. For each function, connect the points with a smooth line or curve.

1. $y = 2x$
   
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

2. $y = x + 4$
   
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

3. $y = 2x - 4$
   
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-4</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>
4. \( y = 2 - 4x \)

\[
\begin{array}{cccccc}
  x: & -2 & -1 & 0 & 1 & 2 & 3 \\
  y: & 10 & 6 & 2 & -2 & -6 & -10
\end{array}
\]

5. \( f(x) = \frac{1}{2}^x \)

\[
\begin{array}{ccccccc}
  x: & 2 & 4 & 6 & 8 & 10 \\
  f(x): & 1 & 2 & 3 & 4 & 5
\end{array}
\]

6. \( f(x) = x^2 \)

\[
\begin{array}{ccccccc}
  x: & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
  f(x): & 9 & 4 & 1 & 0 & 1 & 4 & 9
\end{array}
\]
7. \( f(x) = 2x^2 \)  

\[
\begin{align*}
\text{x:} & \quad -2 & -1 & 0 & 1 & 2 \\
\text{f(x):} & \quad 8 & 2 & 0 & 2 & 8
\end{align*}
\]

8. \( y = -x^2 \)  

\[
\begin{align*}
\text{x:} & \quad -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\text{y:} & \quad -9 & -4 & -1 & 0 & -1 & -4 & -9
\end{align*}
\]

9. \( y = x^2 + 2 \)  

\[
\begin{align*}
\text{x:} & \quad -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\text{y:} & \quad 11 & 6 & 3 & 2 & 3 & 6 & 11
\end{align*}
\]
10. \[ y = \frac{1}{2} x^2 \] 

\[
\begin{array}{cccccccc}
x: & -4 & -2 & -1 & 0 & 1 & 2 & 4 \\
y: & 8 & 2 & 0.5 & 0 & 0.5 & 2 & 8
\end{array}
\]
1. Here is the graph of a function.
   a) Fill out this table for the function:
      \[
      \begin{array}{c|c|c|c|c|c|c|c|c}
        x & 0 & 1 & 2 & 3 & 4 & 5 \\
        \hline
        y & 0 & 5 & 10 & 15 & 20 & 25
      \end{array}
      \]
   b) What happens to \( y \) if \( x \) is doubled?
      \( y \) is doubled
   c) What kind of a function is it?
      directly varying
   d) Write down the formula for this function.
      \( f(x) = 5x \)
   e) What is the value of this function when \( x = 12? \)
      \( f(12) = 60 \)

2. A direct variation is a function that has the form \( y = ax \), and \( a \) is called the “constant of variation”. Answer the following questions about directly varying functions:
   a) What is the value of this function when \( x = 0? \)
      0
   b) What are the coordinates of the point for which \( x = 2? \) For \( x = 5? \)
      For \( x = 2: (2, 2a); \) for \( x = 5: (5, 5a) \)
   c) What happens to the function if \( a \) is negative?
      The slope is negative.
   d) What happens to the function as \( a \) gets larger?
      The slope of the function increases.
   e) What happens to the function if \( a \) is less than 1?
      \( y \) increases more slowly than \( x \) increases (slope is shallow)
   f) What value would \( a \) have to take for the function to be a horizontal line?
      \( a = 0 \) (no slope)

Tell whether the functions represented in the following tables are direct variations, and if they are, write down the constant of variation:

3. \[
\begin{array}{c|c|c|c|c|c|c|c|c}
  x & 0 & 1 & 2 & 3 & 4 & 5 \\
  \hline
  y & 1 & 3 & 5 & 7 & 9 & 11
\end{array}
\]
   No; \( y(x=0) = 1 \)
4. \[ x: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \text{Yes; } a = 1 \]
   \[ y: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

5. \[ x: \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \text{Yes; } a = 7 \]
   \[ y: \quad 7 \quad 14 \quad 21 \quad 28 \quad 35 \]

6. \[ x: \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad \text{Yes; } a = \frac{1}{2} \]
   \[ y: \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

7. \[ x: \quad 1 \quad 2 \quad 3 \quad 4 \quad \text{No; } \frac{4}{1} \neq \frac{2}{3} \]
   \[ y: \quad 4 \quad 3 \quad 2 \quad 1 \]

8. \[ x: \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \text{Yes; } a = -4 \]
   \[ y: \quad -4 \quad -8 \quad -12 \quad -16 \quad -20 \]

9. A certain rubber ball bounces half of the height from which it is dropped. Let \( x \) be the height from which the ball is dropped and let \( y \) be the height that the ball bounces.
   
a) Write the function for the height that this ball bounces.
      \[ y = \frac{x}{2} \]
   b) How high does the ball bounce if it is dropped from a height of 8 feet?
      \[ y \ (x=8 \text{ feet}) = 4 \text{ feet} \]
   c) If the ball bounces 36 inches, what height was it dropped from?
      \[ 36 = \frac{x}{2}; x = 2 \times 36 = 72 \]
   d) What would happen to the constant of variation if the ball was “bouncier”?
      \[ \text{The constant of variation would be larger.} \]
1. Here is the graph of a function.

a) Fill out this table for the function:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>12</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>2.4</td>
<td>2</td>
</tr>
</tbody>
</table>

b) What happens to \( y \) if \( x \) is tripled?
   - It is divided by 3

c) What kind of a function is it?
   - Inversely varying

d) Write down the formula for this function.
   - \( f(x) = \frac{12}{x} \)

e) What is the value of this function when \( x = 12 \)?
   - \( f(12) = 1 \)

f) What value cannot be assigned to \( x \) for this function? Why not?
   - 0; because \( x/0 \) is undefined

g) Can the curve of the graph ever touch the \( y \) axis? What about the \( x \) axis? Why or why not?
   - The curve cannot touch the \( y \) axis, because \( x \) cannot be 0.
   - The curve cannot touch the \( x \) axis, because no matter how large \( x \) gets, \( y \) is still greater than 0.

Remember that a linear function has the form \( y = mx + b \). Make a table for each of the following functions for the values of \( x = 0, 1, 2, 3, 4, \) and 5. Then graph the three functions on the same coordinate axes.

2. \( y = x + 2 \)

3. \( y = x + 3 \)

4. \( y = x + 6 \)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x+2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>x+3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>x+6</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>
5. Answer the following questions about the functions in problems 2-4:

a) What is the slope of each function? 1

b) What is the y-intercept of each function (where the line crosses the y-axis)? 2, 3 and 6

c) What is the effect of increasing the number that is added to x? The y intercept increases (the line moves higher on the graph).

Make a table for each of the following functions for the values of \( x = 0, 1, 2, \) and 3. Then graph the three functions on the same coordinate axes.

6. \( y = 2x + 1 \)

7. \( y = 3x + 1 \)

8. \( y = 4x + 1 \)

\[
\begin{array}{cccc}
\text{x:} & 0 & 1 & 2 & 3 \\
2x + 1: & 1 & 3 & 5 & 7 \\
3x + 1: & 1 & 4 & 7 & 10 \\
4x + 1: & 1 & 5 & 9 & 13 \\
\end{array}
\]

9. Answer the following questions about the functions in problems 6-8:

a) What is the slope of each function? 2, 3 and 4

b) What is the y-intercept of each function? 1

c) What is the effect of increasing the number that is multiplied with \( x \)? The slope increases.
For the given function, fill out the values of \( y \) for the listed values of \( x \).

1. \( y = x + 6 \)
   \[
   \begin{array}{cccccc}
   x: & 0 & 1 & 2 & 3 & 4 & 5 \\
   y: & 6 & 7 & 8 & 9 & 10 & 11
   \end{array}
   \]

2. \( y = 3 - 4x \)
   \[
   \begin{array}{cccccccc}
   x: & -2 & -1 & 0 & 1 & 2 & 3 \\
   y: & 11 & 7 & 3 & -1 & -5 & -9
   \end{array}
   \]

3. \( y = x + 6 \)
   \[
   \begin{array}{ccccccc}
   x: & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y: & 3 & 4 & 5 & 6 & 7 & 8 & 9
   \end{array}
   \]

For the following pairs of numbers in the table, write down the formula for a function that gives the same results. You may write the function using \( y = \) or \( f(x) = \).

4. \[
   \begin{array}{ccccccc}
   x: & 0 & 1 & 2 & 3 & 4 & 5 \\
   y: & 3 & 4 & 5 & 6 & 7 & 8 \\
   \end{array}
   \quad f(x) = x + 3
   \]

For the given function, fill out the values of \( y \) for the listed values of \( x \) and then graph the function on graph paper. Be sure to draw the coordinate axes and label them correctly. For each function, connect the points with a smooth line or curve.

5. \( y = x^2 - 2 \)
   \[
   \begin{array}{cccccccc}
   x: & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   y: & 7 & 2 & -1 & -2 & -1 & 2 & 7
   \end{array}
   \]
6. \( y = (x + 2)^2 \)

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
 x & -4 & -3 & -2 & -1 & 0 & 1 \\
y & 4 & 1 & 0 & 1 & 4 & 9 \\
\end{array}
\]

7. \( y = (x - 2)^2 \)

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
 x & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
y & 9 & 4 & 1 & 0 & 1 & 4 & 9 \\
\end{array}
\]

8. \( y = |x - 2| \)

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c}
 x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
y & 5 & 4 & 3 & 2 & 1 & 0 & 1 \\
\end{array}
\]
9. \[ y = |x| - 2 \]

\[ x: \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \]

\[ y: \quad 1 \quad 0 \quad -1 \quad -2 \quad -1 \quad 0 \quad 1 \]

10. An ice cream shop sells ice cream for 10 cents per ounce. Let \( y \) be the price for a cup of ice cream and let \( x \) be the weight of the ice cream.

\[ y: \text{price of a cup of ice cream} \quad x: \text{weight of ice cream} \]

a) Write a formula for the price of a cup of ice cream as a function of weight.  
\[ y = 10x \]

b) How much will 8 ounces of ice cream cost? How about 16 ounces?
\[ y = 10x = 10 \times 8 = 80 \text{ (cents)} \quad 10 \times 16 = 160 \text{ (cents)} \]

c) Tom spent $2.40 for some ice cream. How much did he buy?
\[ y = 240 = 10x; \quad x = 240/10 = 24 \text{ (ounces)} \]

d) What is the constant of variation in this problem? What happens to it if the price of ice cream increases?

\[ 10 \text{ cents; if price increases, constant of variation increases} \]
Show that the following are rational numbers by writing them as fractions:

1. \(-1 = \frac{-1}{1}\)
2. \(0.444444 \ldots = \frac{4}{9}\)
3. \(0.355 = \frac{355}{1000}\)
4. \(14 = \frac{14}{1}\)
5. \(-5.2 = \frac{-52}{10}\)

Add the correct inequality symbol to compare the following rational numbers:

6. \(\frac{1}{3} > 0.3\)
7. \(-4.1 > -5.01\)
8. \(0.01 > 0.001\)
9. \(\frac{7}{8} > 0.825\)
10. \(\frac{1}{2} < \frac{5}{3}\)
Solve the following problems using absolute values for all the numbers and the correct sign for the problem. Use parentheses where necessary.

Examples:

\[-4 + 2 = -(|-4| - |2|) = -2\]

\[3 - 2 = 3 + (-2) = |3| - |-2| = 1\]

\[-3 \times (-2) = |-3| \times |-2| = 6\]

1. \(5 + 7 = |5| + |7| = 12\)

2. \(-4 + 9 = -(|-4| - |9|) = 5\)

3. \(-10 + (-8) = -(|-10| + |-8|) = -18\)

4. \(15 - 18 = 15 + (-18) = -(|15| + |-18|) = 33\)

5. \(-3 - 1 = -3 + (-1) = -(|-3| + |-1|) = -4\)

6. \(5 - (-2) = 5 + (-(-2)) = 5 + 2 = |5| + |2| = 7\)

7. \(-6 - (-4) = -6 + (-(|-4|)) = -6 + 4 = -(|6| - |4|) = -2\)

8. \(-5 \times (-5) = |-5| \times |-5| = 25\)

9. \(-5 \times 5 = -(|-5| \times |5|) = -25\)

10. \(-6 ÷ 3 = -(|-6| ÷ |3|) = -2\)

11. \(-6 ÷ (-3) = |-6| ÷ |-3| = 2\)
Perform the following approximations:

1. \( \frac{3}{9} \) to the nearest thousandth \( 0.333 \)

2. \( \frac{5}{8} \) to the nearest tenth \( 0.6 \)

3. \(-0.7596\) to the nearest hundredth \( -0.76 \)

4. \(12.49\) to the nearest ten (not tenth) \( 10 \)

5. \(101.9\) to the nearest unit \( 102 \)

6. \(-2.55\) to the nearest tenth \( -2.6 \)

7. \(68,284.2\) to the nearest hundred (not hundredth) \( 68,300 \)

8. \(409,489,521,987\) to the nearest million \( 409,490,000,000 \)

9. \(0.0000005\) to the nearest thousandth \( 0 \)

10. \(-0.12345\) to the nearest ten thousandth \( -0.1235 \)
Graph the following functions for the specified values of the independent variable. You may use your calculator to solve for the value at each point. Be sure to scale your graph appropriately.

1. Graph \( y = x^2 - 0.1 \) for \( x = -0.5, -0.4 \ldots 0.4, 0.5 \)

   ![Graph of \( y = x^2 - 0.1 \)]

2. Graph \( f(x) = \frac{x}{3} \) for \( x = 0, 0.1, 0.2 \ldots 0.5 \)

   ![Graph of \( f(x) = \frac{x}{3} \)]
3. Graph $y = x^2 + x$ for $x = -2, -1, 0, 1, 2$

4. Graph $f(x) = x^2 + x$ for $x = -1, -0.8, -0.6, \ldots, 0.8$ (notice new detail!)

5. Graph $y = 2 - x^2$ for $x = -3, -2 \ldots 3$
Solve and check the following equations by combining like terms and distributing:

1. $3x - 7x + 15 + 6x = 41 \quad x = 13$
2. $2z - 5z + 13 + 9z = 67 \quad z = 9$
3. $3(2x - 5) + 2x = -7 \quad x = 1$
4. $6m + 3(m + 7) = -15 \quad m = -4$
5. $5x - 2(6 - 5x) = 18 \quad x = 2$
6. $4y - (3y + 11) = -11 \quad y = 0$
7. $17 - 7(n - 3) + n = 86 \quad n = -8$
8. $5(7 - x) + 12x = 0 \quad x = -5$
9. $66 = 4(2x - 3) + 2(x + 4) \quad x = 7$
10. $3(a + 2) - (a - 1) = 17 \quad a = 5$
Solve and check the following equation. Write whether the equation is an identity, a conditional equation, or has no solution:

1. $4u = 37 + 4u$  
   \[ \text{no solution} \]

2. $5x + 8 = 7x + 8$  
   \[ \text{conditional; } x = 0 \]

3. $2x - 5 = 3x + 4$  
   \[ \text{conditional; } x = -9 \]

4. $6x + 7 - 2x = 3 + 5x - 9$  
   \[ \text{conditional; } x = 13 \]

5. $3s + 3(1 - s) = s - 17$  
   \[ \text{conditional; } s = 20 \]

6. $4(r + 1) = 6 - 2(1 - 2r)$  
   \[ \text{identity} \]

7. $2[1 - 3(x + 2)] = -x$  
   \[ \text{conditional; } x = -2 \]

8. $3(x - 4) - x = 2(x + 6)$  
   \[ \text{no solution} \]

9. $2(5 - t) + 6t = t + 22$  
   \[ \text{conditional; } t = 4 \]

10. $4(x + 3) = x + 5 + 3x + 7$  
    \[ \text{identity} \]
Solve and check the following equations using your calculator. Approximate the answers to the nearest hundredth. Check your answers after you approximate.

1. \(12x = 5x + 37\) \(x \approx 5.29\)

2. \(57 - 13x = 4x + 8\) \(x \approx 2.88\)

3. \(3.2x = 7.1x + 10.2\) \(x \approx -2.62\)

4. \(0.3c - 8.5 = 1 + 1.7c\) \(c \approx -6.79\)

5. \(4.5 - 7.2x = 3.4x - 49.5\) \(x \approx 5.09\)

6. \(0.3x + 0.4 + 0.5x = 0.6x + 0.7\) \(x = 1.5\)

7. \(0.4x + 0.5 = 0.6x + 0.7 + 0.8x\) \(x = -0.2\)

8. \(3(2.4x + 5) = x + 2.7\) \(x \approx -1.98\)

9. \(6.3 + 1.2s = 4(7.1 - s)\) \(s \approx 4.25\)

10. \(2.4(3.1x + 4.9) = 75.9 + 0.87x\) \(x \approx 9.76\)
Solve the following literal equations for $x$.

1. $5x + t = 17 \quad x = (17 - t)/5$
2. $2x = 4a + 2b \quad x = 2a + b$
3. $x + 3v = 9v \quad x = 6v$
4. $pcx = 2p \quad x = 2/c$
5. $ax/6 = 2b \quad x = 12b/a$
6. $21w/x = 3 \quad x = 7w$
7. $16(x - a) = 4(2a - x) \quad x = 6a/5$

Evaluate the following formulas for the given values of the literal constants:

8. $V = \frac{4}{3} pr^3$, $p = 3.14$, $r = 2 \quad V \approx 33.49$
9. $A = 0.5bh$, $b = 12$, $h = 6 \quad A = 36$
10. $r = \frac{d}{t}$, $d = 500$, $t = 20 \quad r = 25$
Solve the following equations and check your answers. Approximate to the nearest hundredth if necessary. State if the equation is an identity or has no solution.

1. $3x + 8x = 165$ \hspace{1cm} x = 15
2. $4(t + 5) - 3(t + 2) = 14$ \hspace{1cm} t = 0
3. $5(r + 4) - (r - 3) = 5$ \hspace{1cm} r = -4.5
4. $2(4c - 7) = 8c + 14$ \hspace{1cm} no solution
5. $6x + 33 + 5x = 11(x + 3)$ \hspace{1cm} identity
6. $9x = 2x + 58$ \hspace{1cm} x = 8.29
7. $7(2.3 - 0.4x) = x + 1.5$ \hspace{1cm} x = 3.84
8. $31 = 5 - 2(3x + 4) - x$ \hspace{1cm} x = -4.86
9. $5(2x - 3) = 6x + 9$ \hspace{1cm} x = 6
10. $x^2 + x + 3 = x(x + 1)$ \hspace{1cm} no solution
11. $7.4x + 3.8 = 5.9x - 9.7$ \hspace{1cm} x = -9
12. $5.3 - 5.3d = 2.8 + 2.8d$ \hspace{1cm} d = 0.31

Solve the following literal equations for $x$.

13. $ax + by = c$ \hspace{1cm} x = (c - by)/a
14. $a = 2(x + w)$ \hspace{1cm} x = (a/2 - w)
15. $y = mx + b$ \hspace{1cm} x = (y - b)/m
16. $A = \frac{1}{2}xh + \frac{1}{2}yh$ \hspace{1cm} x = $2(A/h - y/2)$
17. $F = \frac{9}{5}x + 32$ \hspace{1cm} x = $5(F - 32)/9$
Evaluate the following formulas for the given values of the literal constants:

18. \( F = \frac{9}{5}C + 32 \), \( C = 40 \) \[ F = 104 \]

19. \( P = 2w + 2l \), \( w = 10, l = 15 \) \[ P = 50 \]

20. \( c = ax^2 + by^2 \), \( a = 3, b = 5, x = 1, y = 2 \) \[ c = 23 \]
Solve the following:

1. A bus is 13 kilometers from town, going away at 80 kilometers per hour (km/h). At the same moment a car leaves town going the same direction as the bus, at the same speed.

   a) Draw a diagram showing town, the bus’ starting point (13 km from town), and the bus and car in appropriate places. Show the bus’s distance from town and the car’s distance from town.

   b) Let \( x \) be the number of hours the car and the bus have been going. Write expressions for each vehicle’s distance from town.

   \[\text{Bus distance from town: } 13 + 80x\]
   \[\text{Car distance from town: } 80x\]

   c) After how long will the bus’s distance from town be twice the car’s distance?

   \[13/80 \text{ hours} = 0.1625 \text{ hours}\]

   d) After how long will the car’s distance from town be 90% of the bus’s distance?

   \[1.4625 \text{ hours}\]

   e) Write an equation stating that both are the same distance from town. Then show that the equation has no solutions. Why is this result reasonable?

   \[13 + 80x = 80x; \quad 13 + 80x - 80x = 80x - 80x; \quad 13 = 0 \text{ (no solution)}\]

   This is reasonable because the vehicles are moving at the same speed, so the car will never catch the bus.
2. A cougar spots a fawn 132 meters away. The cougar starts toward the fawn at a speed of 18 meters per second (m/s). At the same instant, the fawn starts running away at 11 m/s. Let \( t \) be the number of seconds they have been running.

a) Write the definition of \( t \). Then write expressions for the cougar’s and fawn’s distances from the cougar’s starting point after \( t \) seconds.

   \[ \text{Time running: } t \]
   \[ \text{Cougar’s distance from starting point: } 18t \]
   \[ \text{Fawn’s distance from starting point: } 132 + 11t \]

b) How far is the cougar from the fawn after 8 seconds?
   \[ 132 + 11(8) - 18(8) = 76 \]

c) Write an equation stating that the distance between the cougar and the fawn (fawn’s distance minus cougar’s distance) equals 60 meters. Then solve the equation to find out when they are 60 meters apart.
   \[ 132 + 11t - 18t = 132 - 7t = 60; \quad t \approx 10.3 \text{ seconds} \]

d) The cougar has enough energy to run for a total of 17 seconds. Will it catch the fawn before it runs out of energy? Justify your answer.
   \[ 132 - 7(17) = 13; \text{ the cougar will not catch the fawn because he runs out of energy 13 meters before he catches him} \]

3. Fred weighs 187 pounds but is on a diet that makes him lose 1.7 pounds per week. Joe weighs only 93 pounds but is on a diet that makes him gain 0.9 pounds per week.

a) Write an expression representing Fred’s weight after \( x \) weeks and another expression representing Joe’s weight after \( x \) weeks.

Fred’s weight: \( 187 - 1.7x \)
Joe’s weight: \( 93 + 0.9x \)

b) What is each one’s weight after: i) 10 weeks; ii) 1 year?

i) Fred: 170 pounds; Joe: 102 pounds
ii) Fred: 98.6 pounds; Joe: 139.8 pounds

c) After how many weeks will each be the same weight? Show your work.

\[ 187 - 1.7x = 93 + 0.9x \]
\[ 94 = 2.6x \]
\[ x \approx 36.2 \text{ weeks} \]
1. Find all of the rates in the following paragraph:

The Indianapolis 500 is one of the most famous auto races in the United States, with hundreds of thousands of people attending annually. Attendees in 2006 paid from $40 to $150 per ticket plus $20 to $50 per hour to park. Drivers reached speeds of more than 230 mi/hr (354 km/hr) as they raced the 2.5-mile oval track.

- hundreds of thousands of people per year
- $40 per ticket
- $150 per ticket
- $20 per hour parking
- $50 per hour parking
- 230 miles per hour
- 354 km per hour
- 2.5 miles per lap

2. In an average year, Americans spend $40 billion on cosmetics. There are approximately 300,000,000 Americans. How much money on cosmetics are spent for each American? If 120,000,000 Americans are women over the age of 16, how much money on cosmetics is spent for each?

Money for each American: $133.33
Money for each woman over the age of 12: $333.33

3. In 2005, the population of Montana was 0.9 million. The area is 147,000 square miles. What is the population density of Montana?

Approximately 6.1 people per square mile

4. A fast runner can run a half-mile in 2 minutes. Express the average rate in each of these units: (a) miles per minute (b) minutes per mile (c) miles per hour

a) ¼ mile per minute  
   b) 4 minutes per mile  
   c) 15 miles per hour

5. In $t$ minutes a copy machine made $n$ copies. What is the rate in copies per minute? What is the rate in minutes per copy?

Copies per minute: $n/t$  
Minutes per copy: $t/n$

6. A truckload of 56,000 avocados weighs 7 tons. How many avocados per pound? How much does each avocado weigh? (1 ton is 2000 pounds).

Avocados per pound: 4  
Pounds per avocado: ¼

7. If the temperature goes from 70 to 86 degrees in 2 hours what is the change in temperature in degrees per hour?

8 degrees per hour
8. If the temperature goes from 44 to 32 degrees in 5 hours, what is the change in temperature in degrees per hour? \( -2.4 \text{ degrees per hour} \)

9. A steel rod 2 meters long is held at 100 degrees C at one end and 0 degrees C at the other end. What is the rate of change of temperature along the length of the rod? \( 50 \text{ degrees C/meter} \)

10. At the beginning of August Alice weighed 87 pounds. At the end of August Alice weighed 87 pounds. What was her rate of change in weight? \( 0 \text{ pounds per month} \)
1. The graph above shows the Smith family’s checking account balance through the course of a month. Answer the following questions about the data:
   a) In what part of the month is the balance increasing the fastest? *between days 5 and 10*
   b) In what part of the month is the balance decreasing the fastest? *between days 25 and 30*
   c) In what part of the month is the balance staying the same? *between days 10 and 15*

2. Mark went on a hike up a mountain, leaving at 8 AM. On the right is a table showing his altitude throughout the day.
a) Make a graph of Mark’s altitude as a function of time.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Altitude (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 AM</td>
<td>300</td>
</tr>
<tr>
<td>9:00 AM</td>
<td>500</td>
</tr>
<tr>
<td>10:00 AM</td>
<td>1200</td>
</tr>
<tr>
<td>11:00 AM</td>
<td>3300</td>
</tr>
<tr>
<td>12:00 AM</td>
<td>2000</td>
</tr>
<tr>
<td>1:00 PM</td>
<td>1400</td>
</tr>
<tr>
<td>2:00 PM</td>
<td>300</td>
</tr>
</tbody>
</table>
b) What is Mark’s rate of change in altitude between 8 AM and 9 AM? **200 feet/hour**
c) What is Mark’s rate of change in altitude between 9 AM and 11 AM? **1400 feet/hour**
d) What is Mark’s rate of change in altitude between 11 AM and 2 PM? **-1000 feet/hour**
e) In which hour was Mark’s altitude increasing the fastest? **10 AM - 11 AM**
f) In which hour was Mark’s altitude decreasing the fastest? **11 AM - 12 PM**
g) What was Mark’s average rate of change in altitude from 8 AM to 2 PM? **0 feet/hour (because he ended where he started)**

3. A rocket is 1 miles above the earth in 30 seconds and 5 miles above the earth in 2.5 minutes.

   a) What is the rocket’s rate of change in miles per second? **1/30 mile per second**
   b) What is the rocket’s rate of change in seconds per mile? **30 seconds per mile**
   c) What is the rocket’s rate of change in miles per hour? **120 miles per hour**

4. A diver is 20 feet below the surface of the water after 2 minutes and is 80 feet below the surface of the water after 14 minutes. What is her rate of change in feet per minute? **5 feet/minute**

5. George starts the summer in the beginning of June with $175 in his saving account. By the end of June he has $535 in his savings account, and at the end of August he has $1075 in his savings account.

   a) What was the rate of change in George’s savings in dollars per month for the month of June? **$360/month**
   b) Was George saving money at a faster rate in June than he was for the whole summer? **yes, he was saving faster in June**
1. Janet is now 13 years old. Her father Paul is now 36.

(a) Write expressions for each person’s age $x$ years from now.

Janet: $13 + x$
Paul: $36 + x$

(b) Write an equation stating that Paul is twice as old as Janet. Then solve it to find out when Paul is twice as old.

$36 + x = 2(13 + x); \quad x = 10$ years

(c) When is Paul three times as old as Janet? $-3/2$, or 1.5 years ago

(d) Write an equation stating that Paul is exactly as old as Janet. When will this be true? $13 + x = 36 + x; \quad never \ true$

(e) Write an equation stating that Janet’s age is 18 years less than Paul’s age. When will this be true? $13 + x = 36 + x - 18; \quad never \ true$

(f) When will the sum of their ages be: (i) 85 (ii) 36?

(i) $13 + x + 36 + x = 85; \quad x = 18$ years
(ii) $13 + x + 36 + x = 36; \quad x = -6.5$ years (6½ years ago)

2. The ACME Excavation company has a contract to dig a tunnel through Bald Mountain. Crew A starts at the west end and digs at 9 meters per day. Let $t$ be the number of days Crew A has been digging.

a) Write an expression for the number of meters Crew A has dug after $t$ days.

Meters dug by Crew A: $9t$

b) Crew B starts at the east end two days after Crew A and digs at 12 meters per day. In terms of $t$, how many days has Crew B been digging? How many meters have they dug in this same number of days?

$t - 2; \quad 12(t - 2) = 0$ (0 meters)

c) After how many days will both crews have dug the same number of meters?

$12(t - 2) = 9t; \quad t = 8$ days

d) The total length of the tunnel is to be 2000 meters. Write an equation stating this fact. Then solve the equation to find out how many days it takes to dig the tunnel from the time Crew A starts. $12(t - 2) + 9t = 2000; \ 21t = 2024; \ t \approx 96.4$ days

3. A truck passes a highway patrol station going 70 miles per hour (mi/h). When the truck is 10 miles past the station, a patrol car starts after it, going 100 mi/h. Let $t$ be the number of hours the patrol car has been going.

a) Write the definition of $t$. Then write two expressions, one representing the patrol car’s distance from the station and the other representing the truck’s distance from the station after $t$ hours.

$t$: number of hours patrol car has been driving
truck’s distance: $10 + 70t$
patrol car’s distance: $100t$
b) If they continue at the same speeds, who will be farther from the station, and how many kilometers farther, after: i) 10 minutes ii) 30 minutes? 
   i) truck is 5 miles further  ii) patrol car is 5 miles further 
c) At what time \( t \) does the patrol car reach the truck? \( t = \frac{1}{3} \) hours = 20 minutes 
d) Show that the two distances really are the same at the time you calculated in part (c). 
\[ 10 + 70(\frac{1}{3}) = 100(\frac{1}{3}); \quad 10 + 23\frac{1}{3} = 33\frac{1}{3} \text{ (ok)} \]

4. Rich has $100 and spends $3.00 of it per day. Ashley has only $20 but is saving at the rate of $5 per day. Let \( x \) be the number of days that have passed. 

a) Write the definition of \( x \). Then write two expressions, one representing how much Rich has after \( x \) days and the other representing how much Ashley has after \( x \) days. 
   \[ x: \text{number of days that have passed} \]
   \[ \text{Rich has: } 100 - 3x \]
   \[ \text{Ashley has: } 20 + 5x \]
b) Who has more money, and how much, after: i) 1 week ii) 2 weeks 
   i) \( 100 - 3(7) = 79; \quad 20 + 5(7) = 55; \quad \text{Rich has more ($24 more)} \)
   ii) \( 100 - 3(14) = 58; \quad 20 + 5(14) = 90; \quad \text{Ashley has more ($32 more)} \)
c) After how many days will each have the same amount of money? Write and solve an equation to find this number of days. 
   \[ 100 - 3x = 20 + 5x; \quad x = 10 \]
d) Show that each actually does have the same amount of money after the number of days you calculated in part (c). 
\[ 100 - 3(10) = 20 + 5(10); \quad 100 - 30 = 20 + 50 \text{ (ok)} \]

5. If Jack’s savings account balance increases from $212 to $758 in two months, what is his rate of savings per month? What is his rate of savings per year? 
   \[ \$273/\text{month} = \$3276/\text{year} \]

6. For the following items at the grocery store, what is the unit price for each item, and how many of each do you get for $1? 
   a) One dozen eggs for $3.30 \[ \$0.275/\text{egg}; \text{approximately } 3.6 \text{ eggs/dollar} \]
   b) 8 ounces of cheese for $1.20 \[ \$0.15/\text{ounce}; \text{approximately } 6.7 \text{ ounces/dollar} \]
   c) 64 ounces of orange juice for $4.00 \[ \$0.0625 \text{ per ounce}; 16 \text{ ounces/dollar} \]

7. At a baseball game, Big Joe ate 15 hotdogs in 90 minutes. 
   a) What was his rate of eating hotdogs in hotdogs per minute? \[ 1/6 \text{ hotdogs/minute} \]
   b) How many minutes did he take for each hotdog? \[ 6 \text{ minutes/hotdog} \]
   c) What was his rate in hotdogs per hour? \[ 10 \text{ hotdogs/hour} \]
8. Farmer Bob is selling tomatoes at his roadside vegetable stand. In the morning he has $50 in his till and 1,872 tomatoes in his bin. By afternoon he has $492 in his till and 546 tomatoes in his bin.

a) What is the bin’s rate of change in tomatoes per dollar? 3 tomatoes/dollar
b) What is the till’s rate of change in dollars per tomato? $0.33½/tomato
c) If Farmer Bob was at his stand from 11 AM to 1 PM, what is his selling rate in tomatoes per hour? 663 tomatoes/hour
d) If Farmer Bob was at his stand from 11 AM to 1 PM, what is his selling rate in dollars per hour? $221/hour

9. Mr. and Mrs. Jones are 35 years old, and they would like to retire when they are 75 years old. They have $50,000 in their bank account, and they believe that they will need $1,000,000 to retire comfortably.

a) How much will they need to save each year in order to meet their retirement goal? $23,750
b) How much will they need to save each month in order to meet their retirement goal? approximately $1979.17